Lessons 45-46

Math 70 11.7 & 11.8 Maximum and Minimum Problems with Quadratic Functions
and GC 32: Use GC to find max/min.

Objectives:

1) Use the vertex formula to find the vertex of a quadratic function $f(x) = ax^2 + bx + c$.
   a. *x*-coordinate $h = -\dfrac{b}{2a}$, *y*-coordinate $k = f(h)$
2) Identify whether a quadratic function $f(x) = ax^2 + bx + c$ or $f(x) = a(x-h)^2 + k$ has a maximum or minimum value.
3) Use the vertex formula to find the maximum or minimum value of a quadratic function.
   a. A quadratic function has only one – either a maximum or a minimum, not both.
   b. The max or min occurs at the vertex.

CAUTION: Vocabulary for the maximum/minimum and its location are important because it tells you which coordinate of the vertex is the answer to the question.

   c. The maximum or minimum value is the *y*-coordinate of the vertex, $y = k = f(h)$, because it is the maximum function value or minimum function value.
   d. The location of the maximum or minimum is the *x*-coordinate of the vertex. $x = h = -\dfrac{b}{2a}$

4) Use the vertex formula to solve maximum/minimum word problems.

NOTE: The question "find the maximum or minimum" is a general problem. In Math 70, we only do this for quadratic functions, so in Math 70, the key words "Maximum" or "Minimum" connect to "Vertex Formula".

Practice and Examples

1) Does $h(t) = -16t^2 + 10t + 100$ open upward or downward? Does $h(t) = -16t^2 + 10t + 100$ have a maximum or minimum? Why?

2) An object is thrown upward from the top of a 100-ft cliff. Its height h in feet above the base of the cliff after t seconds is given by $h(t) = -16t^2 + 10t + 100$. Find the maximum height of the object and the number of seconds it took to reach that max height.

3) Does $C(x) = 2x^2 + -800x + 92000$ open upward or downward? Does $C(x) = 2x^2 + -800x + 92000$ have a maximum or minimum? Why?

4) The cost C of manufacturing x bicycles at Holladay's Production Plant is given by the function $C(x) = 2x^2 + -800x + 92000$
   a. Find the number of bicycles that must be manufactured to minimize the cost.
   b. Find the minimum cost.

5) The Utah Ski Club sells calendars to raise money. The profit P, in cents, from selling x calendars is given by the equation $P(x) = 360x - x^2$.
   a. Find how many calendars must be sold to maximize profit.
   b. Find the maximum profit.

6) Find two numbers whose sum is 60 and whose product is as large as possible. [Hint: Let x and 60-x be the two positive numbers. Their product can be described by the function $f(x) = x(60 - x)$.]

7) The length and width of a rectangle must have a sum of 40 cm. Find the dimensions of the rectangle that will have the maximum area.

8) Methane is a gas produced by landfills, natural gas systems, and coal mining that contributes to the greenhouse effect and global warming. Projected methane emissions in the US can be modeled by the quadratic function $f(x) = -0.072x^2 + 1.93x + 173.9$, where f(x) is the amount of methane produced in million metric tons and x is the number of years after 2010.
   a. According to this model, what will US emissions of methane be in 2019? (Round to two decimal places.)
   b. Will this function have a maximum or a minimum? Explain.
   c. In what year will methane emissions in the US be at their maximum/minimum? Round to the nearest whole year.
   d. What is the level of methane emissions for that year? (Use your rounded answer from part c.) (Round this answer to 2 decimal places.)

Extras:

9) Find two numbers whose sum is 11 and whose product is a large as possible.
10) Find two numbers whose difference is 10 and whose product is as small as possible.
11) Find two numbers whose difference is 8 and whose product is as small as possible.
12) The length and width of a rectangle must have a sum of 50. Find the dimensions of the rectangle that will have maximum area.

13) Explore finding vertex by GC tables
+ Review problems on vertex, x-ints, graphing

Math 70

① Does $h(t) = -16t^2 + 10t + 100$ open upward or downward?
Does $h(t) = -16t^2 + 10t + 100$ have a maximum or minimum?
Why?

$$h(t) = -16t^2 + 10t + 100$$

is like $\quad f(x) = -16x^2 + 10x + 100 \quad$ where f was changed to h and x was changed to t.

$a = -16$ is negative
so parabola opens $\boxed{\text{downward}}$

A downward parabola  has a $\boxed{\text{maximum value}}$

because the vertex is at the top of all other points (x,y) on the parabola so the y-coordinate of the vertex is greater than all other y-coordinates.

② An object is thrown upward from the top of a 100-ft cliff. Its height h in feet above the base of the cliff after t seconds is given by $h(t) = -16t^2 + 10t + 100$. Find the maximum height of the object and the number of seconds it took to reach that max height.

Maximum occurs at the vertex.

If the function is written $f(x) = a(x-h)^2 + k$, we can read the vertex from the function.
But if the function is written $f(x) = ax^2 + bx + c$, we find the vertex using the vertex formula.

Vertex Formula for $f(x) = ax^2 + bx + c$	
x-coordinate of vertex	$= \dfrac{-b}{2a}$
y-coordinate of vertex	$= f\left(\dfrac{-b}{2a}\right)$

* Memorize.

↑ Want to know where this came from?
Use completing-the-square... to see this done, look at the end of this lesson.

Find the vertex of $h(t) = -16t^2 + 10t + 100$

x-coord $\approx$ t coordinate $= \dfrac{-b}{2a} = \dfrac{-10}{2(-16)} = \dfrac{5}{16} = .3125$

y-coord $\approx$ h coord $= h\left(\frac{5}{16}\right) = -16\left(\frac{5}{16}\right)^2 + 10\left(\frac{5}{16}\right) + 100$
$f\left(\frac{-b}{2a}\right)$ $\quad h\left(\frac{-b}{2a}\right)$

$= \dfrac{1625}{16} = 101.5625$

The vertex is $\left(\frac{5}{16}, \frac{1625}{16}\right)$ or $(.3125, 101.5625)$.

The maximum height is the h (or y) coordinate of the vertex.

$\boxed{\text{max height } 101.5625 \text{ ft}}$

The time the object reaches the max height is the t (or x) coordinate of the vertex.

$\boxed{\text{time of max height } 0.3125 \text{ sec}}$.

③ Does $C(x) = 2x^2 - 800x + 92000$ open upward or down? Does it have a max or min? Why?

$C(x) = ax^2 + bx + c$
$\uparrow$
$2x^2 - 800x + 92000$

$a = 2$ is positive

$\boxed{\begin{array}{l} a > 0 \\ \text{opens up} \\ \text{has a minimum} \end{array}}$

④ The cost C of manufacturing x bicycles is given by $C(x) = 2x^2 - 800x + 92000$.

a) Find the number of bicycles to minimize cost.

b) Find minimum cost.

These key words, in Math 70, mean "find the vertex".

Math 70

vertex formula: $x = \frac{-b}{2a}$

$= \frac{-(-800)}{2(2)}$

$= \frac{800}{4}$

x coordinate of vertex $= \boxed{200 \text{ bicycles}}$ a)

y-coordinate of vertex $= C(200)$

$= 2(200)^2 - 800(200) + 92000$

$= \boxed{\$12000 \text{ minimum cost}}$ b)

(5) Profit P, in cents, from selling x calendars is $P(x) = 360x - x^2$.

a) How many calendars to maximize profit?

b) Find max profit.

P(x) is not written in standard form!

$P(x) = -x^2 + 360x \implies a = -1$ neg. ∩

$b = 360$

$c = 0$.

vertex formula

x-coordinate of vertex $= \frac{-b}{2a} = \frac{-360}{2(-1)} = \boxed{180 \text{ calendars}}$

y-coordinate of vertex $= P(180) = -(180)^2 + 360(180)$

$= 32400$ cents.

$= \boxed{\$324.00}$

Math 70

(6) Find two numbers whose sum is 60 and whose product is as large as possible.

$$\left.\begin{array}{l} A = \text{1st number} \\ B = \text{2nd number} \end{array}\right\} \quad \text{sum is } 60 \Rightarrow A + B = 60$$

$$\text{product} \Rightarrow AB$$

Too many variables, and it's not a quadratic!

Solve $A+B=60$ for either variable

$$B = 60 - A$$

Substitute into product

$$\text{Product} = A(60-A)$$

distribute

$$\text{Product} = 60A - A^2$$

$$P(A) = -A^2 + 60A \qquad \text{or} \qquad f(x) = -x^2 + 60x$$

$a = -1 < 0$ has maximum.

vertex formula

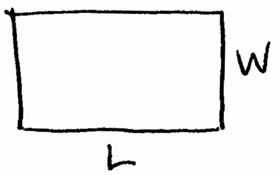
$$A = \frac{-b}{2a} = \frac{-60}{2(-1)} = 30$$

$$B = 60 - A \implies 60 - 30 = 30$$

The two numbers are $\boxed{\text{30 and 30.}}$

Math 70

7 The length and width of a rectangle must have a sum of 40cm. Find the dimensions of the rectangle that will have the maximum area.



Sum $= L+W = 40$

area $= L \cdot W$

Solve $L+W=40$ for a variable (doesn't matter)

$$L = 40 - W$$

substitute into area

$$\text{area} = (40-W)\cdot W$$

distribute

$$\text{area} = 40W - W^2$$

$$A(W) = -W^2 + 40W \qquad \text{or} \qquad f(x) = -x^2 + 40x$$

$a = -1 < 0$ $\frown$ maximum.

Find vertex using vertex formula

$$W = \frac{-b}{2a} = \frac{-40}{2(-1)} = \boxed{20\text{ cm} = W}$$

$$L = 40 - 20 = \boxed{20\text{ cm} = L}$$

Math 70

8. Methane emissions $f(x) = -0.072x^2 + 1.93x + 173.9$
where $f(x)$ = amount of methane, in million metric tons
and x = # years after 2010.

a) Emissions in 2019?

$$\begin{array}{r} 2019 \\ -\ 2010 \\ \hline 9 = x \end{array}$$

Find $f(9) = -0.072(9)^2 + 1.93(9) + 173.9$
$= \boxed{185.438 \text{ million metric tons}}$

b) Maximum or minimum?

$a = -0.072 < 0$ $\frown$ $\boxed{\text{maximum.}}$ $\boxed{a<0}$

c) What year at maximum? (nearest whole year)

vertex formula $\frac{-b}{2a} = \frac{-1.93}{2(-0.072)}$

$= 13.40277778$
≈ 13 years $= \boxed{2023}$

d) What is the level of methane emissions that year? Use rounded answer from c, then round to 2 decimal places.

$f(13) = -0.072(13)^2 + 1.93(13) + 173.9$

$= 186.822$

$\approx \boxed{186.82 \text{ million metric tons}}$

Math 70

Extras

9 2 numbers, sum is 11, product as large as possible

$A+B=11 \Rightarrow B=11-A$

$AB=\max$

$P(A)=A(11-A)$

$=11A-A^2$

$=-A^2+11A$ or $f(x)=-x^2+11x$

vertex $\frac{-b}{2a}=\frac{-11}{2(-1)}=\frac{11}{2}=5.5$

$B=11-5.5=5.5$ $\boxed{5.5 \text{ and } 5.5}$ or $\boxed{\frac{11}{2} \text{ and } \frac{11}{2}}$

10 difference is 10, product as small as possible.

$A-B=10 \Rightarrow A=B+10$

$A\cdot B=\min.$

$P(B)=(B+10)\cdot B$

$=B^2+10B$ or $f(x)=x^2+10x$

vertex formula $\frac{-b}{2a}=\frac{-10}{2(1)}=-5=B.$

$A=B+10 \Rightarrow A=-5+10=5$

$\boxed{5 \text{ and } -5}$

11 difference is 8, product as small as possible.

$A-B=8 \Rightarrow A=B+8$

$AB=\min$

$P(B)=B(B+8)$

$=B^2+8B$ or $f(x)=x^2+8x$

Vertex formula $\frac{-b}{2a}=\frac{-8}{2(1)}=-4=B$

$A=-4+8=4$

$\boxed{4 \text{ and } -4}$

Math 70

(12) The length and width of a rectangle must have a sum of 50. Find dimensions of rectangle with max area.

sum = $L + W = 50$

area = $L \cdot W$

Solve $L + W = 50$ for either variable

$$W = 50 - L$$

substitute

$$\text{area} = L(50 - L)$$

distribute

$$\text{area} = 50L - L^2$$

standard form

$$\text{area} = -L^2 + 50L \qquad \text{or} \qquad f(x) = -x^2 + 50x$$

vertex formula

$$\frac{-b}{2a} = \frac{-50}{2(-1)} = 25 = L$$

$$W = 50 - L \Rightarrow W = 50 - 25$$

$$\boxed{25 \times 25}$$

M70

when finding vertex by GC

- students who do tables get different (wrong) answers depending on Δx.

(13) Ex. $y = -16x^2 + 103x$

$y(103/32) = 10609/64 = 165.765625$

$\frac{-b}{2a} = \frac{-103}{2(-16)} = 3.21875$

$\boxed{\text{Correct} \rightarrow \text{using MAX} \quad (3.21875,\ 165.765625)}$

WRONG - USING TABLE:

using $\Delta x = 1$: max $(3, 165)$ wrong!

$\Delta x = .5$ max $(3, 165)$ wrong!

$\Delta x = .1$ max $(3.2, 165.76)$ wrong!

$\Delta x = .01$ max $(3.22, 165.77)$ wrong!

$\Delta x = .001$ max (

every value from 3.213 to 3.225 rounds to 165.77

$(3.219, 165.765624)$

wrong!

M70

Review

① Rewrite $f(x) = -3x^2 - 12x - 11$

a) using CTS

b) using vertex formula.

a) $y = -3x^2 - 12x - 11$

$y + 11 = -3(x^2 + 4x \quad)$

$\# = \frac{4}{2} = 2$ ← goes in squared factor later

$\#^2 = 2^2 = 4$ ← add inside ( )

$y + 11 - 3(4) = -3(x^2 + 4x + 4)$

$y + 11 - 12 = -3(x+2)^2$

$y - 1 = -3(x+2)^2$

$y = -3(x+2)^2 + 1$

$\boxed{f(x) = -3(x+2)^2 + 1}$

b) $h = \frac{-b}{2a} = \frac{-(-12)}{2(-3)} = \frac{12}{-6} = -2$

$k = f(-2) = -3(-2)^2 - 12(-2) - 11$

$= -3(4) + 24 - 11$

$= 1$

$f(x) = a(x-h)^2 + k$

$\boxed{f(x) = -3(x+2)^2 + 1}$

② Solve $-3x^2 - 12x - 11 = 0$.
Compared to ①, what have we found? $\boxed{\text{set } y=0 \text{ means x-ints}}$

<u>Method 1</u>: Quadratic formula

$a = -3 \quad b = -12 \quad c = -11$

easier $3x^2 + 12x + 11 = 0$

$a = 3 \quad b = 12 \quad c = 11$

(2) con't

$$x = \frac{-12 \pm \sqrt{12^2 - 4(3)(11)}}{2(3)}$$

$$x = \frac{-12 \pm \sqrt{12}}{6}$$

$$x = \frac{-12}{6} \pm \frac{2\sqrt{3}}{6}$$

$$\boxed{x = -2 \pm \frac{\sqrt{3}}{3}}$$

← These are real numbers. The quadratic function has 2 (decimal) x-intercepts.

Method 2: Completing the square.

$$\frac{-3x^2}{-3} \frac{-12x}{-3} \frac{-11}{-3} = \frac{0}{-3}$$

$$x^2 + 4x + \frac{11}{3} = 0$$

$$x^2 + 4x = -\frac{11}{3}$$

$\# = \frac{4}{2} = 2$ ← use in squared factor later

$\#^2 = 2^2 = 4$ ← add to both sides now

$$x^2 + 4x + 4 = -\frac{11}{3} + 4$$

$$(x+2)^2 = \frac{1}{3}$$

$$(x+2) = \pm\sqrt{\frac{1}{3}} = \pm\frac{\sqrt{1}}{\sqrt{3}} = \pm\frac{1}{\sqrt{3}\cdot\sqrt{3}}\cdot\sqrt{3} = \pm\frac{\sqrt{3}}{3}$$

$$\boxed{x = -2 \pm \frac{\sqrt{3}}{3}}$$

Math 70

Review

(3) Graph $f(x) = -\frac{1}{2}x^2 + 3x - \frac{5}{2}$

vertex $h = \frac{-b}{2a} = \frac{-3}{2(-\frac{1}{2})} = \frac{-3}{-1} = 3$

$k = f(3) = -\frac{1}{2}(3)^2 + 3(3) - \frac{5}{2}$

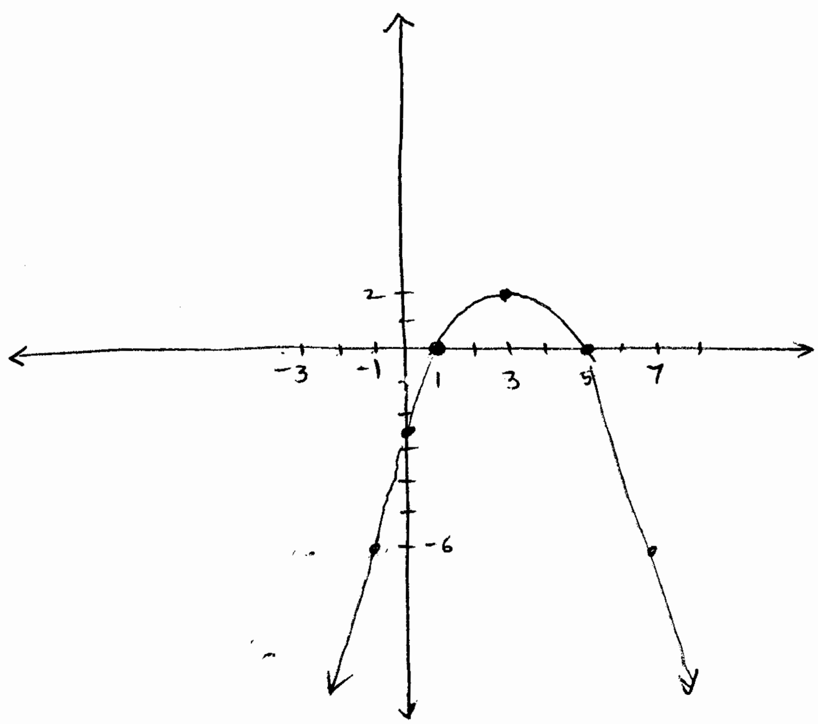
$= 2$

vertex $(3, 2)$

$a = -\frac{1}{2}$ opens downward, wider than standard.

Use GC table for graphing

x	$f(x)$	
-2	-10.5	
$-$	-6	✓
0	-2.5	
1	0	✓
2	1.5	
3	2	✓
4	1.5	
5	0	✓
6	-2.5	
7	-6	✓
8	-10.5	



Either change y-scl to 0.5 on graph
or choose points marked ✓ in the table.

Rev 8-8-13

Name ______________________________

Date ______________________________

TI-84+ GC 32: Maximum or Minimum of a Quadratic Function

Objectives: Review the direction of a parabola and the sign of its leading coefficient
Identify which functions will have maxima (minima) from leading coefficients
Recognize the maximum (or minimum) function value as the y-coordinate of vertex
Find approximate maximum and minimum values using the GC

A Quadratic Function has a squared x, or a degree 2 term, and passes the Vertical Line Test.
General Form: $f(x) = ax^2 + bx + c$, where a, b, and c are constants, $a \neq 0$.
Standard Form: $f(x) = a(x-h)^2 + k$, where a, h, and k are constants, $a \neq 0$, and (h,k) are the coordinates of the vertex.
If a>0, the parabola opens upward. If a<0, the parabola opens downward. a is called the leading coefficient.
The vertex of a quadratic function can be found using the vertex formula: Find the x-coordinate first $x = -\frac{b}{2a}$, then evaluate the function at the x-value found: $y = f\left(-\frac{b}{2a}\right)$.

1) What is the leading coefficient of $f(x) = 0.5x^2 - 8x + 35$? Does $f(x) = 0.5x^2 - 8x + 35$ open upward or downward?

2) Find the exact vertex of $f(x) = 0.5x^2 - 8x + 35$.

3) What is the leading coefficient of $f(x) = -2x^2 + 20x - 52$? Does $f(x) = -2x^2 + 20x - 52$ open upward or downward?

4) Find the exact vertex of $f(x) = -2x^2 + 20x - 52$.

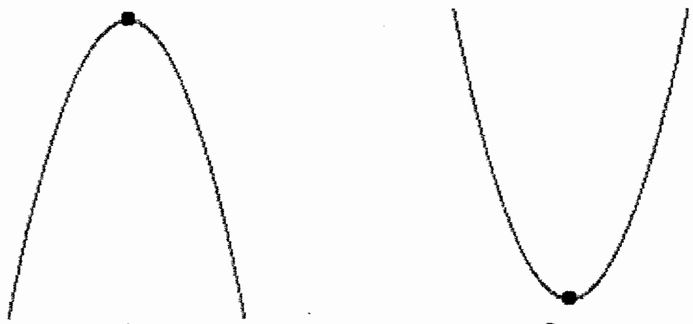
5) What is the leading coefficient of $f(x) = -x^2 + 10x - 4.75$? Does $f(x) = -x^2 + 10x - 4.75$ open upward or downward?

6) Find the exact vertex of $f(x) = -x^2 + 10x - 4.75$.

When graphing, we plot points (x, y) where $y = f(x)$. This means that the y-coordinate is the function value. When we seek the <u>maximum</u> value of a function, we are seeking the largest y-coordinate on the graph, or the highest point. When we seek the <u>minimum</u> value of a function, we are seeking the smallest y-coordinate on the graph, or the lowest point.

Sometimes a question will ask you for the x-coordinate at which the maximum (or minimum) occurs.

When the graph of a quadratic function opens <u>downward</u>, the y-coordinates have a <u>maximum</u> value, which occurs at the vertex. The y-coordinates do not have a minimum value, because the graph continues infinitely down, off the page. "Infinity" is not a number, so it's not a minimum.



$a<0$, down

$a>0$, up

When the graph of a quadratic function opens <u>upwards</u>, the y-coordinates have a <u>minimum</u> value, which occurs at the vertex. The y-coordinates do not have a maximum value, because the graph continues infinitely up, off the page. "Infinity" is not a number, so it's not a maximum.

7) Can a quadratic function have both a maximum value AND a minimum value?

8) Can a quadratic function have NEITHER a maximum value nor a minimum value?

9) If the leading coefficient is a positive number, does the quadratic function have a maximum or a minimum? If the leading coefficient is a negative number, does the quadratic function have a maximum or a minimum?

10) Is the maximum value of a quadratic function the x-coordinate or the y-coordinate? Is the minimum value of a quadratic function the x-coordinate or the y-coordinate?

11) What do we call the point were the maximum value of a quadratic function occurs? What do we call the point were the minimum value of a quadratic function occurs?

12) Does $f(x) = 0.5x^2 - 8x + 35$ have a maximum or minimum value?

To find the approximate minimum value using the GC, use Minimum, in the CALC menu, which is 2nd TRACE.
To find the approximate maximum value using the GC, use Maximum, in the CALC menu, which is 2nd TRACE.
Both of these functions are three-step calculations.
Step 1: Identify a value of x which is less than (to the left of) the vertex (Left Bound)
Step 2: Identify a value of x which is greater than (to the right of) the vertex (Right Bound)
Step 3: Identify a value of x which is close to the vertex (Guess)

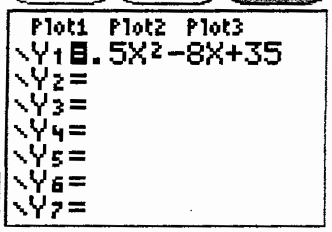
If you cannot see the maximum or minimum value in the GC window, the GC can't find it.

13) Find the approximate minimum value of $f(x)=0.5x^2-8x+35$ using the GC.

Enter the function in the Y= menu and graph in a standard window:

Y= CLEAR . 5 X,T,θ,n x^2 − 8 X,T,θ,n + 3 5

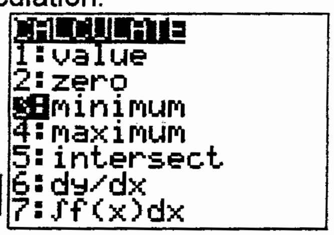
ZOOM 6



Make sure you can see the vertex on the screen – if not, adjust the window.

Begin the Minimum calculation:

2nd TRACE 3



Step 1: Move to the left of the vertex to mark the left bound:
The coordinates of the cursor are displayed at the bottom of the screen.

If x is too small, use ⟩. If x is too large, use ⟨

Look before you press enter – make sure the cursor is left of the vertex! ENTER

Step 2: Move to the right of the vertex to mark the right bound:

⟩ Look before you press enter – make sure the cursor is right of the vertex! ENTER

Step 3: Use last point as the guess:

ENTER

Write down the minimum value – the y-coordinate of the vertex.

14) Find the approximate vertex of $f(x) = -2x^2 + 20x - 52$ using the GC.

Enter the function in the Y= menu and graph in a standard window:

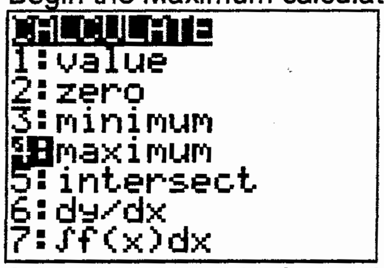
[Y=] [(−)] [2] [X,T,Θ,n] [x^2] [+] [2] [0] [X,T,Θ,n] [−] [5] [2]

[ZOOM] [6]

[2nd] [TRACE] [4]

Make sure you can see the vertex on the screen – if not, adjust the window.

Begin the Maximum calculation:



Step 1: Move to the left of the vertex to mark the left bound:
The coordinates of the cursor are displayed at the bottom of the screen.

If x is too small, use [▸]. If x is too large, use [◂]

Look before you press enter – make sure the cursor is left of the vertex! [ENTER]

Step 2: Move to the right of the vertex to mark the right bound:

[▸] Look before you press enter – make sure the cursor is right of the vertex! [ENTER]

Step 3: Use last point as the guess:

[ENTER]

Write down the maximum value – the y-coordinate of the vertex.

15) Does $f(x) = -x^2 + 10x - 4.75$ have a maximum or minimum value? Find the approximate value using your GC.

16) Does $f(x) = 1.2x^2 - 11x - 26$ have a maximum or minimum value? Find the approximate value using your GC. Round to the nearest hundredth.

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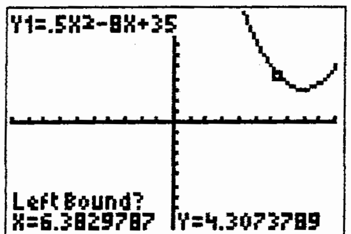
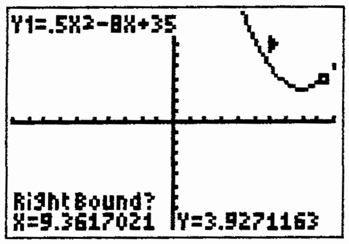
1) a=0.5, up
2) V(8,3)

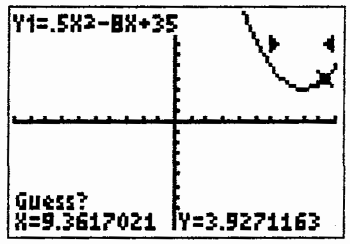
3) a=-2, down
4) V(5,-2)

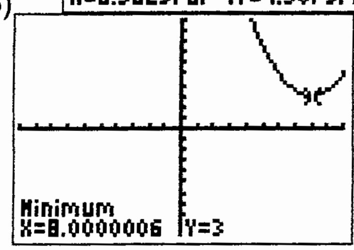
5) a=-1, down
6) (5,20.25)

7) no
8) no
9) a positive (upward), minimum; a negative (downward) maximum
10) y-coordinate, y-coordinate
11) vertex, vertex
12) a=.5, positive (upward), has minimum

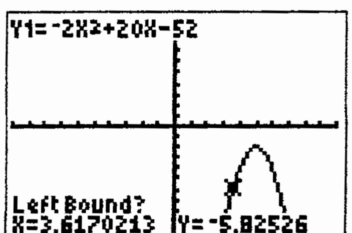
13)


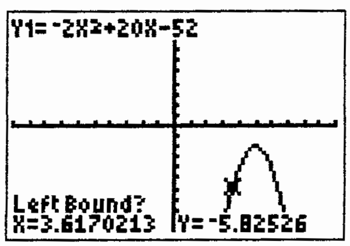


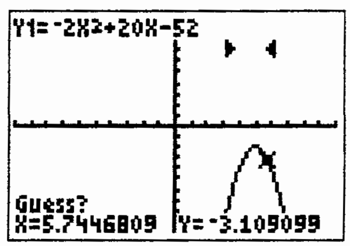


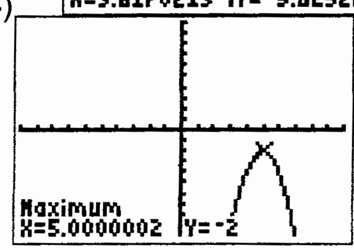


minimum value 3.

14)


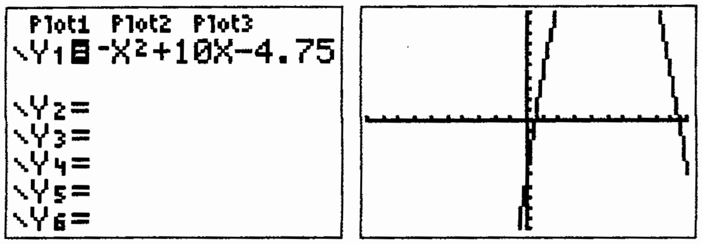


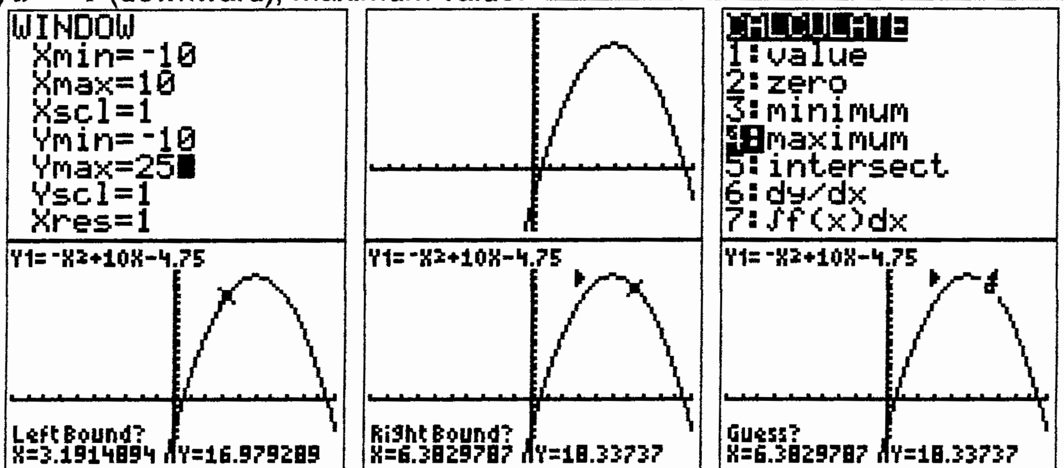


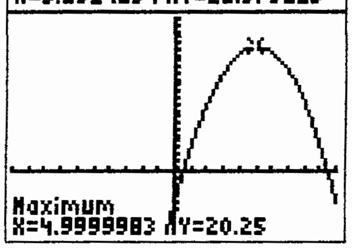


maximum value -2

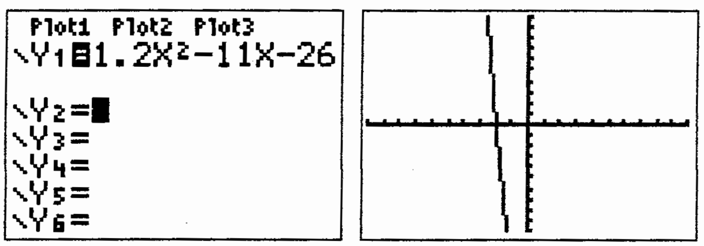
15) $a = -1$ (downward), maximum value.



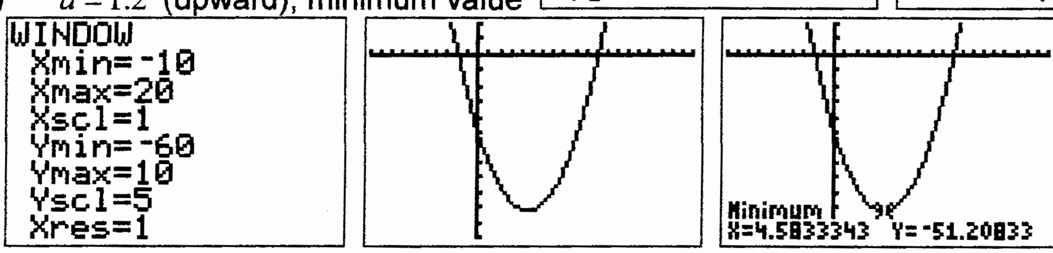




maximum 20.25



16) $a = 1.2$ (upward), minimum value



minimum value to nearest hundredth: -51.21

Rev 8-8-13

Name ______________________________

Date ______________________________

TI-84+ GC 29: Graph to Paper: X-intercepts of Quadratic Functions

Objectives: Review quadratic functions, vertex, general and standard forms of the equation
Find the vertex of a quadratic function written in standard form
Find the exact x-intercepts of a quadratic function using algebra
Find approximate x-intercepts of a quadratic function using Zero on the GC
Draw a graph from a non-standard GC window on paper with labeled axes

A Quadratic Function has a squared x, or a degree 2 term, and passes the Vertical Line Test.

General Form: $f(x) = ax^2 + bx + c$, where a, b, and c are constants, $a \neq 0$.

Standard Form: $f(x) = a(x-h)^2 + k$, where a, h, and k are constants, $a \neq 0$, and (h,k) are the coordinates of the vertex.

If a>0, the parabola opens upward. If a<0, the parabola opens downward.

The vertex of a quadratic function can be found using the vertex formula: Find the x-coordinate first $x = -\frac{b}{2a}$, then evaluate the function at the x-value found: $y = f\left(-\frac{b}{2a}\right)$.

To write general $f(x) = ax^2 + bx + c$ from standard $f(x) = a(x-h)^2 + k$: FOIL and simplify.

To write standard $f(x) = a(x-h)^2 + k$ from general $f(x) = ax^2 + bx + c$: complete the square.

When graphing a quadratic function, you must graph the vertex, plot at least four additional points, and neatly draw the parabola. Instructions may also require you to graph x-intercepts. A parabola is a rounded shape – there should not be a point at the vertex.

The axis of symmetry of a quadratic function is a vertical line through the vertex which divides the graph in two mirror halves. Its equation is $x = -\frac{b}{2a}$, same as the x-coordinate of the vertex.

1) Find the vertex of $f(x) = 3(x-20)^2 + 5$.

2) Find the vertex of $f(x) = -2x^2 + 4x - 5$.

Recall: The x-intercepts of any equation or function are points where the graph crosses the x-axis, having y-coordinate 0. To find the x-intercepts algebraically, set y=0 and solve for x. (If given f(x) notation, replace it by 0.)
X-intercepts must be real numbers. If the algebraic result is imaginary, there are no x-intercepts.

3) Find the exact x-intercepts of $f(x)=3x^2-2x-1$ using algebra.

4) Find the exact x-intercepts of $f(x)=3(x-2)^2+5$ using algebra. (Hint: Use the square root property.)

To find approximate x-intercepts using the GC, use Zero, in the CALC menu, which is 2nd TRACE.

5) Find the approximate x-intercepts of $f(x)=3(x-2)^2-5$ using the GC. Round to nearest tenth.

Step 1: Enter the function in the Y= menu and graph in a standard window:

[Y=] [CLEAR] [3] [(] [X,T,θ,n] [−] [2] [)] [x^2] [−] [5] [ENTER]

[ZOOM] [6]

Step 2: Begin the Zero calculation

[2nd] [TRACE] [2]

Step 3: Move to the left of the left x-intercept to mark the left bound:

[◀] Look before you press enter – make sure the cursor is left of the x-intercept! [ENTER]

Step 4: Move to the right of the left x-intercept to mark the right bound:

[▶] Look before you press enter – make sure the cursor is right of the x-intercept! [ENTER]

Step 5: Use last point as the guess:

[ENTER]

Repeat for the other x-intercept.

[2nd] [TRACE] [2] [▶] [▶] [▶] [▶] [ENTER] [▶] [▶] [▶] [▶]

[ENTER] [ENTER]

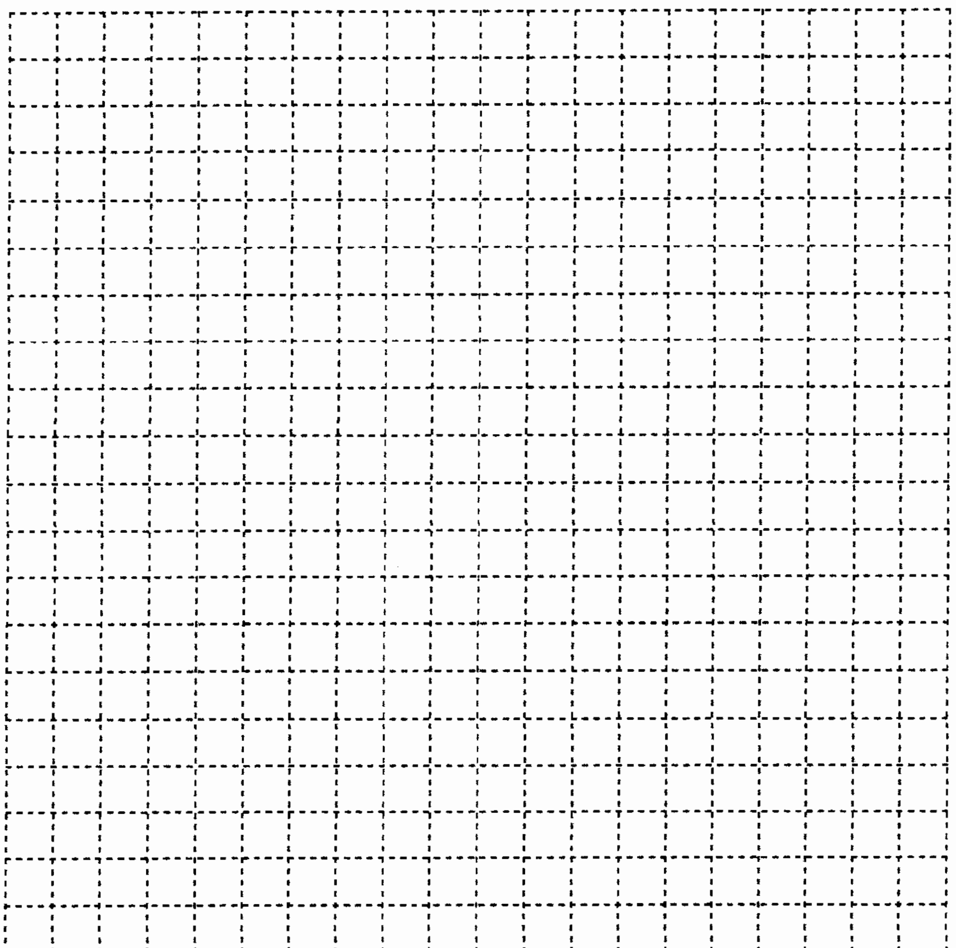
6) Find the approximate x-intercepts of $f(x)=2x^2-5$ using the GC. Round to nearest hundredth.

To find additional points on the parabola, select values of x that are close to the x-coordinate of the vertex and evaluate the function for these values.

7) If graphing the function $f(x)=3(x-20)^2+5$, what is the vertex? Make a table on your GC and use it to find two additional integer pairs you could plot on the graph. (Hint: Make TblStart = x-coordinate of the vertex.)

8) Neatly graph $f(x)=3(x-20)^2-5$. Plot and label the x-intercepts.

Step 1: Find the vertex. (Use previous result.)
Step 2: Find exact or approximate x-intercepts using algebra or the GC
Step 3: Find two other points with integer coordinates. (Use previous result.)
Step 4: Look at all the points and decide location of the x-and y-axes.
Step 5: Determine scale for the axes. Draw and label axes.
Step 6: Plot the vertex and draw the axis of symmetry.
Step 7: Plot the points and use symmetry. Neatly connect points in a rounded parabola.



x values	y values

9) Neatly graph $f(x) = 3x^2 - 2x - 1$. Plot and label the x-intercepts.

Sometimes integer values of x are hard to find. You found the vertex in a previous question. (Hint: $\frac{1}{3}$ appears a lot. Use it! You can make a GC table using $\Delta Tbl = 1/3$)

x values	y values

(Hint: Use scale = $\frac{1}{3}$ for the value of one tick mark on the x-axis, and also on the y-axis.)

Rev 8-8-13

Name ______________________________

Date ______________________________

TI-84+ GC 33 Number of x-Intercepts and Zoom-In To Find Difficult x-Intercepts

Objectives: Use leading coefficient and location of vertex to identify number of x-intercepts
Use GC and knowledge of quadratic functions to find difficult x-intercepts

For each of the following scenarios, how many x-intercepts does the quadratic function $f(x) = ax^2 + bx + c$ have? [Hint: Use the sign of the leading coefficient to imagine or draw the direction the graph opens.]

1) a>0, vertex is below the x-axis.

2) a>0, vertex is above the x-axis

3) a>0, vertex is on the x-axis

4) a<0, vertex is below the x-axis

5) a<0, vertex is above the x-axis

6) a<0, vertex is on the x-axis

7) If the vertex is on the x-axis, what is its y-coordinate?

8) Use the vertex formula to find the vertex of $y = x^2 + 6.3x + 9.9$.

9) Is the vertex you found in the previous question above, below, or on the x-axis?

10) What is the leading coefficient of $y = x^2 + 6.3x + 9.9$?

11) How many x-intercepts does this function have?

12) Find the approximate x-intercepts of $y = x^2 + 6.3x + 9.9$ using the GC. Round to the nearest thousandth, if necessary.

Step 1: Graph the equation:

[Y=] [X,T,θ,n] [x^2] [+] [6] [.] [3] [X,T,θ,n] [+] [9] [.] [9]
[ENTER] [ZOOM] [6]

Step 2: Zoom in on an x-intercept.

Step 2a: Use option 2 under the ZOOM menu: [ZOOM] [2]

Step 2b: Move the ZOOM cursor close to an x-intercept using [◀] or [▶]. The location of the ZOOM cursor will become the new center of the graph.

Step 2c: Press [ENTER].

Step 2d: Since this graph is a hard one, you'll probably have to ZOOM in two more times before you can see the x-intercepts clearly.

ZOOM 2 (or) ENTER ZOOM 2 (or) ENTER

Step 3: Use Zero (in the CALC menu, which is the 2nd function of TRACE) to find approximate value of x-intercept.

Step 3a: Begin the Zero calculation: 2nd TRACE 2

Step 3b: Press (or) to move the cursor to the left bound of the x-intercept.

The press ENTER.

Step 3c: Press) to move the cursor to the right bound of the x-intercept. ENTER.

Step 3d: Press ENTER for the guess.

Result: ____________________

Step 4: Repeat steps 2 and 3 to locate the other x-intercept. Start in the standard window using ZOOM 6.

Result: ______________________

13) Find all the x-intercepts of $f(x) = -2.35x^2 + 0.927x$. Round to the nearest thousandth if necessary.

14) Find all the x-intercepts of $f(x) = \frac{1}{7}(x - 4.173)^2 + 0.005$. Round to the nearest thousandth if necessary.

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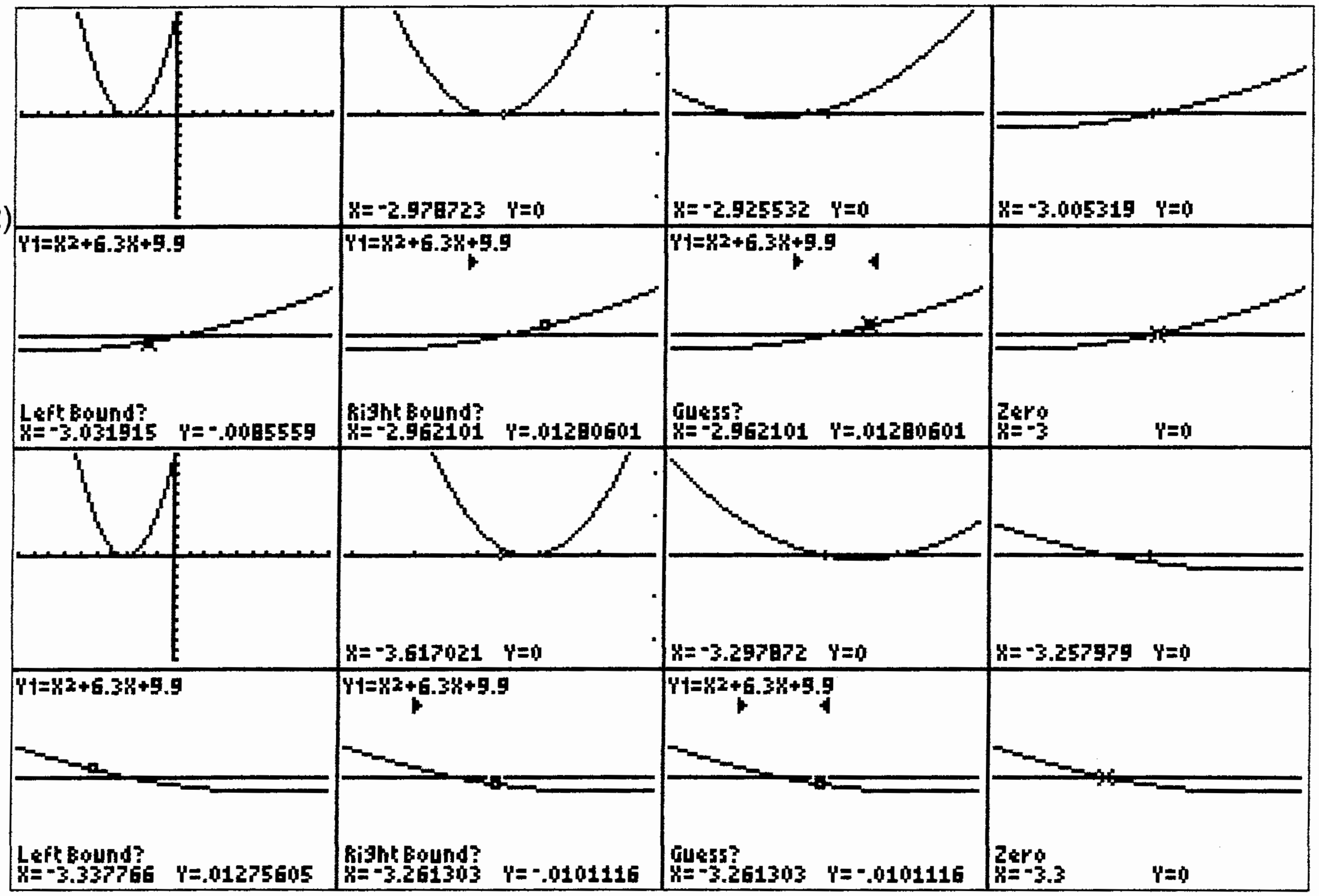
TI-84+ GC 33 Number of x-Intercepts and Zoom-In To Find Difficult x-Intercepts Solutions, page 3

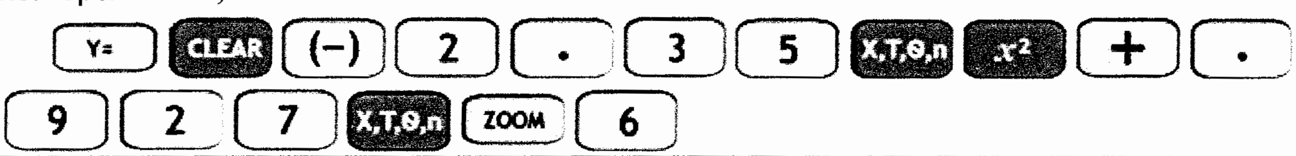
1) 2
2) 0
3) 1
4) 0
5) 2
6) 1
7) y=0 for any point on the x-axis
8) $V(-3.15,-0.0225)$
9) below the x-axis
10) a=1, a>0
11) two x-intercepts

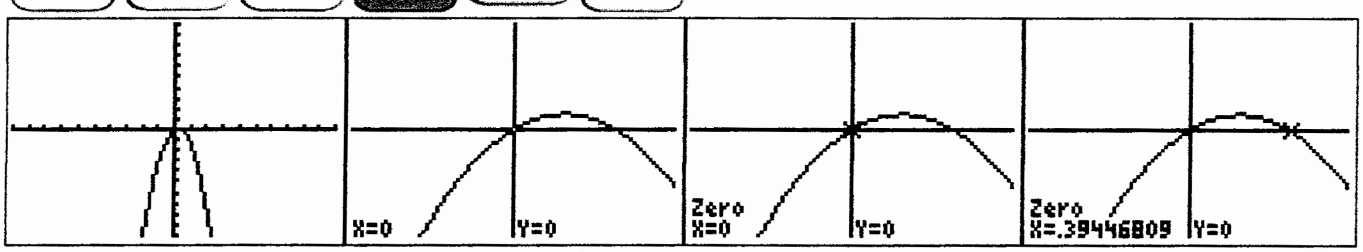
12)



x-intercepts $x=-3,-3.3$

13)





x-intercepts $x=0$, $x \approx 0.394$ These can be found easily using algebra.

14) The vertex is (4.173, .005), above the x-axis. $a=\frac{1}{7}>0$. It has no x-intercepts.

Where does the vertex formula come from?

Use $f(x) = ax^2 + bx + c$ and complete the square.

$$f(x) = a\left(x^2 + \frac{b}{a}x \qquad\right) + c$$ factor out a.

$$\# = \frac{b}{a}\cdot\frac{1}{2} = \frac{b}{2a}$$

$$\#^2 = \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$$

$$a\cdot\#^2 = a\cdot\frac{b^2}{4a^2} = \frac{b^2}{4a}$$

complete the square

$$f(x) = a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + c - \frac{b^2}{4a}$$

$$f(x) = a\left(x + \underbrace{\frac{b}{2a}}_{x}\right)^2 + \underbrace{\left(c - \frac{b^2}{4a}\right)}_{k}$$

Remember: subtract h $y = a(x-h)^2 + k$

vertex formula $x = \frac{-b}{2a}$

$$y = c - \frac{b^2}{4a}$$

$$y = f\left(\frac{-b}{2a}\right) = a\left(\underbrace{\frac{-b}{2a} + \frac{b}{2a}}_{0}\right)^2 + c - \frac{b^2}{4a}$$

$$y = c - \frac{b^2}{4a}$$

This gives a formula for k also, but most people can't remember it and just plug $\frac{-b}{2a}$ in to get y.